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THE RIGOROUS ANALYSIS OF STRONGLY COUPLED STEP DISCONTINUITIES IN MICROSTRIP

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ABSTRACT

A rigorous formulation for the analysis of strongly coupled step discontinuities in boxed microstrip is presented. Use is made of a variational method in which the transverse E field at the discontinuity is expanded in a suitable set of basis functions. Strongly coupled steps are analysed using the concept of "localised" and "accessible" modes and making use of a multi-port network model. Results are presented for the double step discontinuity.

1. INTRODUCTION

The accurate analysis of microstrip discontinuities, including coupling effects, is an important requirement in the design of filters, stepped impedance transformers and other microwave components. This is especially true in the design of microwave integrated circuits where adjustments after fabrication are very difficult or impossible to carry out.

The currently available methods for use in the computer aided design of microwave components which involve discontinuities, e.g. [1-2] rely heavily on quasi-static approximations which are only correct in the limit of low frequency and which suffer significant error as the frequency increases.

Other methods have been used to produce a frequency dependent solution to the microstrip discontinuity problem. These, however, have either used an approximate model e.g. [3], or have been rigorous but have not addressed the problem of strongly coupled discontinuities e.g. [4].

The formulation presented herein makes use of the Galerkin variational method for deriving the generalised S parameters of a single step discontinuity. This lends itself to the treatment of strongly interacting discontinuities by means of the concept of "accessible" and "localised" modes [5]. Each discontinuity is treated as a multiport device, each port corresponding to an accessible mode. Likewise the microstrip which connects successive discontinuities is modelled as a set of transmission lines, each carrying one accessible mode. In this way the coupling can be accurately accounted for.

The E field at the discontinuity is expanded both in the set of microstrip modes each side of the step in order to construct the Green's function, and also in a suitable set of basis functions appropriate to the step itself in order to discretise the integral equation. An essential prerequisite is knowledge of the higher order modes of the spectrum, including complex ones, in as much as up to 100 modes may be required in constructing a properly convergent Green's function for the scattering problem [6].

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2. THE FORMULATION OF THE SINGLE STEP DISCONTINUITY

Most formulations of the microstrip step discontinuity make use of the equivalent circuit shown in Fig. 2. This model, however, suffers from the disadvantages that it is only correct in the limit of low frequency, and that as it stands it cannot be used to model strongly coupled steps.

The formulation presented here uses the model shown in Fig. 3. The step is represented by a multi-port device with frequency dependent S parameters. Each port on the model corresponds to an accessible mode, that is a mode which does not decay to negligible levels by the time it reaches the next discontinuity. Since there is no incident localised mode, these modes can be considered to be terminated in their characteristic impedance. Combination of these S matrices by standard methods makes possible the characterisation of cascades of strongly coupled discontinuities.

Referring to the plan of Fig. 1 we start from the continuity equations for the transverse E and H fields.

$$\sum_n (a_n^{(1)} + b_n^{(1)}) E_n^{(1)} = \sum_n (a_n^{(2)} + b_n^{(2)}) E_n^{(2)} = \epsilon \quad (1)$$

$$\sum_n (a_n^{(1)} - b_n^{(1)}) H_n^{(1)} = -\sum_n (a_n^{(2)} - b_n^{(2)}) H_n^{(2)} + z \times J \quad (2)$$

where

- the coefficients "a" represent the incident wave amplitudes
- the coefficients "b" represent the scattered wave amplitudes
- the superscripts (1) and (2) refer to the regions defined in Fig. 1.

Since we are at liberty to choose the amplitudes of each incident mode, we can, by applying the superposition principle, express the total E field at the step as:

$$\epsilon(r) = \sum_{p=1}^{\alpha} a_p^{(1)} \epsilon_p(r) + \sum_{p=1}^{\beta} a_p^{(2)} \epsilon_q(r) \quad (3)$$

where α and β are the numbers of accessible modes in regions (1) and (2) respectively. $q = p + \alpha$.

By means of algebraic manipulation, we arrive at the following variational expressions for the elements of the S matrix. These are solved using Galerkin's method and expanding the unknown E field in terms of a suitable set of basis functions $\{\phi_n\}$.

$$\frac{\langle \epsilon_t | g | \epsilon_p \rangle}{\langle \epsilon_t, H_p^{(1)} \rangle \langle \epsilon_p, H_t^{(1)} \rangle} = \frac{1}{S_{pt} - \delta_{pt}} \quad (4)$$

and the Green's function g is given by:

$$2g(r|r') = \sum_{n=1}^{\infty} \frac{H_n^{(1)}(r) H_n^{(1)}(r')}{\langle E_n^{(1)}, H_n^{(1)} \rangle} + \frac{H_n^{(2)}(r) H_n^{(2)}(r')}{\langle E_n^{(2)}, H_n^{(2)} \rangle} \quad (5)$$

Clearly it is necessary to truncate the summation of equation 5 after a finite number of terms. It is important, however, that a sufficient number of terms be taken so that convergence is achieved. It has been found that about 100 terms are required. An efficient method of calculating them has previously been described [6].

In addition, the selection of the set of basis functions is crucial to the success of the method. Unfortunately, as the corner of the step discontinuity is a three-dimensional edge, there is no available closed form expression for the fields in the vicinity of the metal. Various sets of basis functions which satisfy the boundary conditions on the box and the strip but which do not incorporate any beliefs concerning the form of the edge condition, have been tried. In the majority of cases convergence has not been satisfactory. So far best results have been obtained using the field patterns of the microstrip containing the wider of the two strips. These vector functions meet the boundary conditions but do not have the correct singularity at the corner. The ratio of E_x to E_y in these vector functions is left as a parameter in the variational formulation. If this was not done, then the higher order modes, excited by the discontinuity, would be orthogonal to the basis functions and would not contribute to the sum in equation 5. It is well-known that the above is the root cause of "relative convergence". To our best knowledge it is the first time that this problem has been circumvented.

Figure 8 shows the convergence of the phase of S_{12} for the single step discontinuity shown in Figure 1. It can be seen that convergence is achieved after about 7 basis functions, with a strip width ratio of 4:1. This is a much smaller matrix that needs handling in the mode matching method.

In addition, numerical experiments have been carried out using the modes of the wider strip multiplied by an expression of the form:

$$\left\{ \left\{ \frac{a-w}{2} \right\}^2 - x^2 \right\}^\mu$$

where a is the box width, w is the wider strip width, and μ is a parameter which is chosen to achieve best convergence. The multiplication was carried out by taking the convolution of the Fourier transform of the above expression expressed in terms of Bessel functions, with the previously calculated Fourier components of the modal fields. By this means it was hoped to improve on the results obtained by using the unchanged microstrip modes as basis functions by bringing the edge behaviour closer to what it really is. Results for various values of μ were obtained but the convergence showed no improvement over that achieved using the modes with the scalar modification discussed above.

3. NETWORK FORMULATION OF MULTIPLE DISCONTINUITIES

Cascades of multiple discontinuities are modelled by connecting corresponding single step models as shown in Fig. 4. The first and last of these networks have all but the dominant modes terminated in their characteristic impedances. Once the S matrices for each discontinuity are known and the propagation coefficients of the intervening microstrip for each accessible mode is known, then the overall S matrix can be calculated using standard methods (e.g. [1]).

4. RESULTS FOR THE DOUBLE STEP DISCONTINUITY

The above method has been applied to the double step discontinuity, the plan of which is shown in Figure 5. At frequencies of 3 GHz and 7 GHz the input VSWR was calculated as a function of the length of the step. The results are shown in Figures 6 and 7. Here we have the results of taking just one accessible mode, i.e. assuming the steps have negligible coupling and the results of taking two accessible modes. In addition the results using quasi-static formulae are shown. It can be seen that at 7 GHz the calculated resonant length is noticeably changed when the second accessible mode is included, thus indicating a significant amount of coupling. At 3 GHz there is no noticeable effect in including the second accessible mode, indicating that at this frequency the coupling between the steps is negligible.

5. CONCLUSION

A variational method for the analysis of step discontinuities in microstrip has been presented. By treating one such step as a multiport network, we can take account of the scattered higher order modes when they are of significant amplitude at a neighbouring discontinuity. Thus we are able to analyse cascades of closely spaced step discontinuities. The results for the single step are in agreement with quasi-static formulae at low frequency and with other rigorous formulations at higher frequencies. Results for the VSWR of a double step discontinuity have been presented which show the effect of including the effect of the scattered higher order modes.

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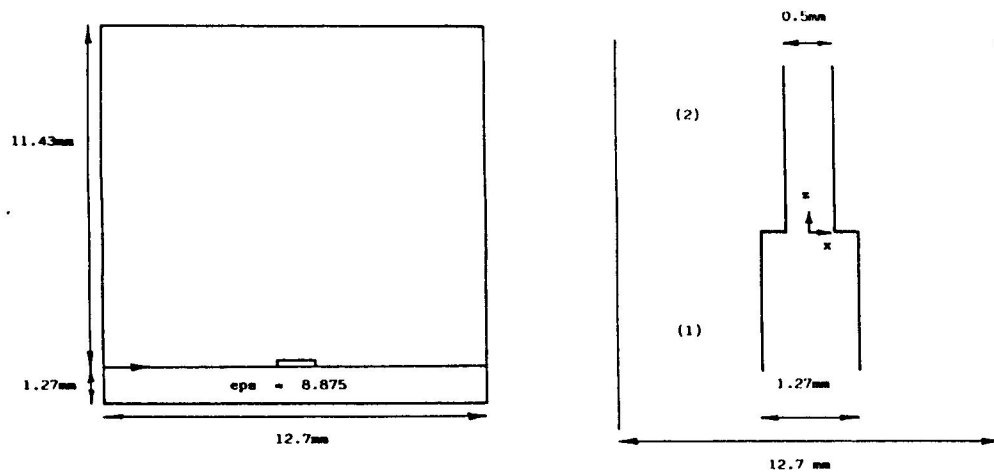


Fig. 1. Plan of step discontinuity

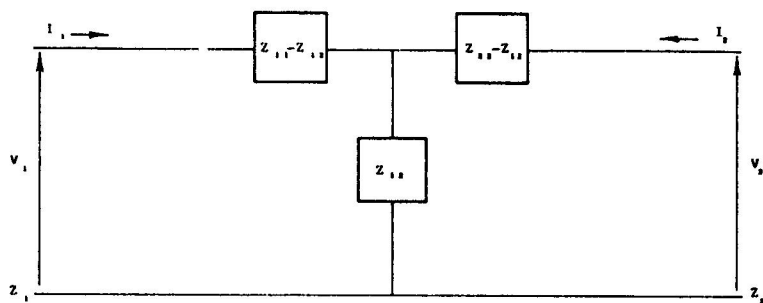


Fig. 2. Equivalent circuit of microstrip step discontinuity.

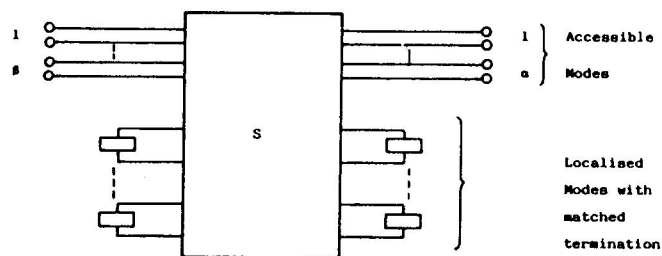


Fig. 3. Network Model of single step.

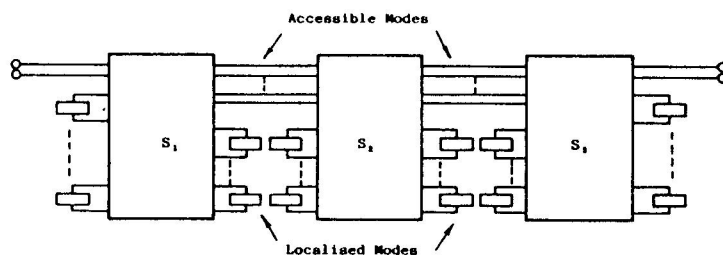


Fig. 4. Network model of coupled steps.

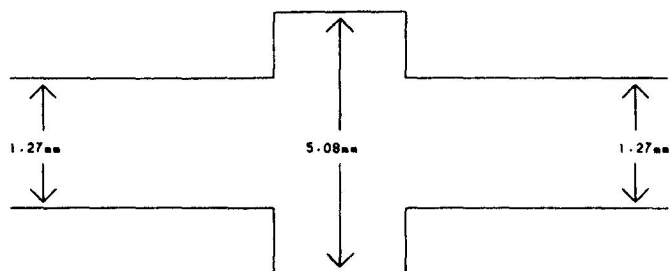


Fig 5 Geometry of a double step discontinuity

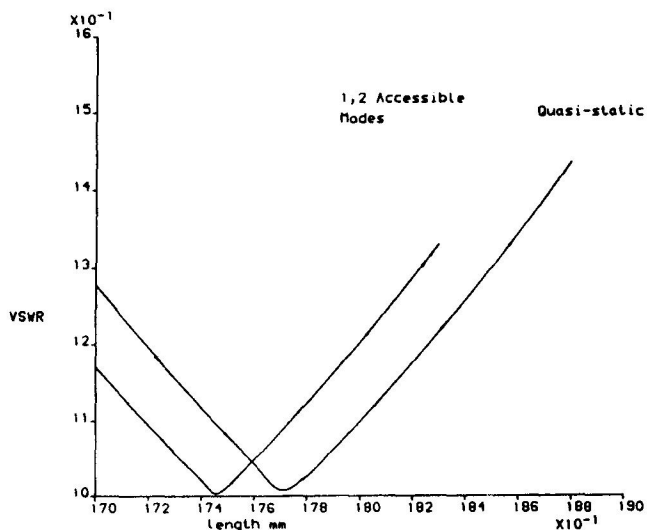


Fig. 6 - VSWR of a double step discontinuity, 30GHz
a=12.7mm d=1.27mm h=10.43mm eps=10 w1=w3=1.27mm w2=5.08mm

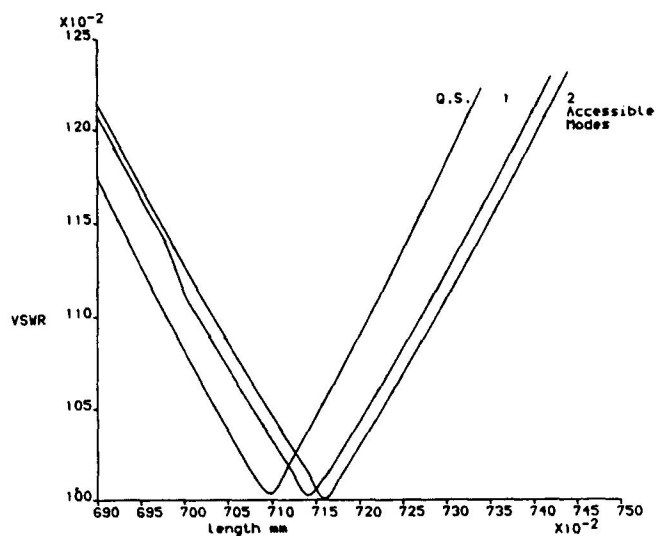


Fig. 7 - VSWR of a double step discontinuity, 76GHz
a=12.7mm d=1.27mm h=10.43mm eps=10 w1=w3=1.27mm w2=5.08mm

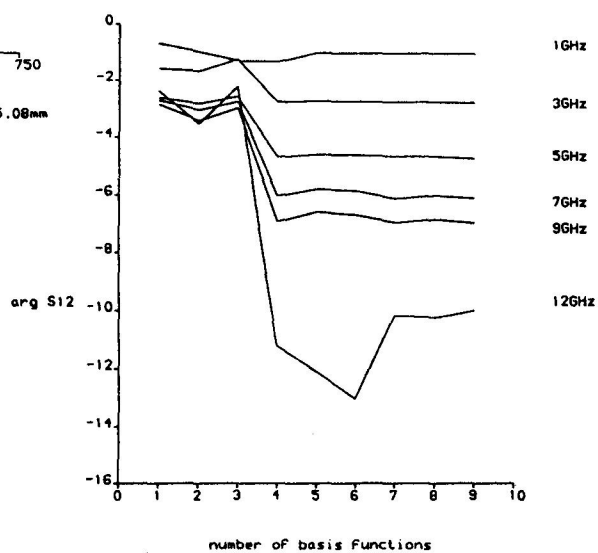


Fig 8 - Convergence of S12 as basis functions increased